Orchestrating Organizational Politics: Baron and Ferejohn Meet Tullock*

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Abstract

This paper examines the optimal organizational rules that govern the process of dividing a fixed surplus. The process is modeled as a sequential multilateral bargaining game with costly recognition. The designer sets the voting rule—i.e., the minimum number of votes required to approve a proposal—and the mechanism for proposer recognition, which is modeled as a biased generalized lottery contest. We show that for diverse design objectives, the optimum can be achieved by a dictatorial voting rule, which simplifies the game into a standard biased contest model.

Keywords: Multilateral Bargaining; Costly Recognition; Contest Design

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1 Introduction

Organizations—whether firms, academic institutions, political parties, etc.—are political structures that "operate by distributing authority and setting a stage for the exercise of power (Zaleznik, 1970)." Organizational power grants individuals preferred access to scarce resources or broader oversight of vital activities (Black, Hollingsworth, Nunes, and Simon, 2022; Pfeffer, 1993), which, in turn, motivates efforts to acquire power and leverage it to influence key decision-making within an organization. Such dynamics are commonplace and inherent in organizational life. For instance, executives strive to ascend the hierarchical ladders to positions that offer significant authority over corporate agendas. Politicians campaign for electoral nominations or party leadership (Mattozzi and Merlo, 2015). Members of blockchain-technology-enabled decentralized autonomous organizations (DAOs) devise and contribute proposals to mobilize community resources for funding their projects.[1]

In this paper, we delve into the crafting of rules that govern this process and regulate the interactions between individuals within an organization. We identify two critical institutional elements that underpin these organizational rules: (i) the organization's evaluation and promotion mechanism, which selects key executives and allocates power and authority, and (ii) the protocol for collective decision-making that either expands or limits the executive's power to influence the distributive outcome. The institutional setting defines how power is to be acquired, whom is to be awarded the power, and the value placed on that power for each individual. The choice of institutional elements addresses the ultimate challenge for effective organization design, which calls for "balancing control mechanisms with incentives that drive performance" (Galbraith, 1973), and speaks to the enduring debate over centralized vs. decentralized power structure within organizations (Mintzberg, 1973; Kotter, 1985; Alonso, Dessein, and Matouschek, 2008).

For this purpose, we examine organizational interactions through the lens of a multilateral sequential bargaining process (Baron and Ferejohn, 1989). A pool of agents—e.g., business units inside a firm, academic departments of a school, politicians within a political party, or community members of a DAO—divide a fixed amount of resources. One agent proposes a plan for resource sharing, and must secure a minimum number of favorable votes from peers to approve and implement the plan. The conventional wisdom of the literature on multilateral bargaining holds that the proposer enjoys a disproportionately large share, which drives the contest prior to the bargaining: Agents expend costly efforts to compete for the right to propose, which reflects the costly recognition mechanism à la Yildirim (2007) and Ali (2015).

We endogenize the rules of this process. A designer sets (i) the bargaining protocol and

[1] Readers are referred to https://en.wikipedia.org/wiki/Decentralized_autonomous_organization.

(ii) the recognition mechanism—i.e., the rules that govern the selection of proposer. The former is defined by a k-majority rule, whereby k denotes the minimum number of favorable votes (including the proposer) required for approval. This depicts the inclusiveness of the collective decision-making process and the limit of the power or influence the proposer enjoys. The latter converts agents' efforts into their recognition probabilities. By varying the recognition mechanism, the designer can effectively bias the competition in favor of certain contenders, and thereby tilt the playing field and reshape agents' incentives. For instance, a preferred candidate in a company's succession process is often assigned a significant position—e.g., president or COO—that enhances their visibility to board members, and a politician competing for party leadership can be endorsed by powerful party elites (Cross, 2013).

The governance of an organization could address diverse interests. We allow for a general objective that accommodates concerns about effort profiles and agents' recognition probability profile. The designer values agents' productive effort contributions, so the objective function (weakly) increases with each agent's effort.[2] Consider, for instance, a political activist's services that contribute to the party's electoral influence and the quality of its platform. Similarly, a corporate executive's performance not only advances their career but also adds to the firm's value. The objective also accounts for concerns about the ex ante power distribution—i.e., the recognition probability profile.

The game is intuitive by nature, but its analysis poses a technical challenge. To vie for recognition, agents weigh their potential payoffs from winning—i.e., being recognized—against those from losing. This payoff differential effectively functions as the *prize spread* that motivates their efforts in the contest. In contrast to a standard contest with a prize value $k \geq 2$, the prize spread in this game is endogenously determined. With a non-dictatorial voting rule, the winner offers a subset of peers—namely, agents in his winning coalition—their equilibrium continuation values in the dynamic bargaining process to secure their votes; a loser, on the other hand, receives his equilibrium continuation value if included in a winning coalition, or nothing if excluded. The prize spreads—which depend on agents' continuation values—motivate their efforts, while agents' efforts determine their recognition probabilities, the formation of each agent's winning coalitions, and, ultimately, their continuation values in equilibrium.

The endogeneity, together with agents' heterogeneity in contest technologies, effort cost functions, and/or patience levels, complicates the analysis and differentiates the game from traditional bargaining or contest models. A change in either institutional element—the vot-

[2]The conventionally assumed objectives in the literature on optimal contest design—such as total effort maximization and the expected winner's effort maximization—are both special cases.

ing rule or the recognition mechanism—triggers complex effects and precludes the possibility of unambiguous comparative statics.

We establish equilibrium existence in the game in Section 3, although uniqueness is elusive. Due to the aforementioned complications, the game does not yield a closed-form equilibrium solution, which renders the usual implicit programing approach ineffective. We elaborate on the economic nature of these institutional variables in Sections 4.1 and 4.2. We develop a technique to identify the optimum: We demonstrate that when the designer can set both the voting rule and the recognition mechanism, the optimum always requires a *dictatorial voting rule* with $k = 1$, although the specific form of recognition mechanism depends on the particular environmental factors. That is, a proposal is accepted with the consent of only the proposer, which results in an agent's capturing the entire surplus once recognized and receiving nothing otherwise. The game thus reduces to a standard static contest. Notably, this result holds even if the designer's sole objective is the fair distribution of bargaining power ex ante. The universal optimality of the dictatorial voting rule raises a further question: Suppose the designer can flexibly adjust the recognition mechanisms. Does a less inclusive voting rule—i.e., a smaller k—always improve the value of the objective function? We demonstrate that this does not hold in general. However, our analysis shows that when agents are relatively impatient—i.e., when their patience levels are limited by an upper bound—monotonicity is preserved. The intuition of our results will be discussed in greater depth in Section 4 after we formally present the results.

Our analysis not only provides novel theoretical insights but also generates significant policy and managerial implications. The results offer an alternative perspective on the endogenous formation of centralized internal power structures. We demonstrate that bargaining power can be optimally allocated to a single individual, even when the organization prefers equal distribution of recognition probabilities. Specifically, our insights illuminate long-standing debates on the organization of political parties, such as whether a political party subject to electoral accountability requires intra-party democracy (IPD). The ascent of blockchain-based DAOs provides another relevant context, in which voting protocols and proposing eligibility are critical for governance through smart contracts. These implications will be further explored in Section 4.4.2.

Link to the Literature Our paper adds to the literature on multilateral bargaining by providing a comprehensive analysis to endogenize the bargaining protocol and the proposer recognition mechanism.

An extensive body of literature has emerged from the canonical framework established by Baron and Ferejohn (1989) to explore the process of distributive politics—e.g., Merlo and

Wilson (1995, 1998); Banks and Duggan (2000); Eraslan (2002); Eraslan and Merlo (2002, 2017); Diermeier and Fong (2011); Diermeier, Prato, and Vlaicu (2015, 2016); Ali, Bernheim, and Fan (2019); and Evdokimov (2023). The majority of this literature assumes that the proposer is exogenously and randomly selected from the agents.

A small but growing strand of the literature considers the selection of the proposer to be an integral part of the political process, and examines the endogenous formation of bargaining protocols. Yildirim (2007) models the process to select proposers as a contest in which agents exert costly effort to gain power, and pioneers the integration of a contest model (generalized Tullock contest) with a multilateral bargaining game to endogenize the recognition mechanism. Yildirim (2010) compares total effort and distributive outcomes between persistent and transitory recognition procedures, and Ali (2015) models the recognition process as an all-pay auction.

Our paper extends the effort to incorporate recognition mechanisms in a holistic distribution process and models the recognition process as an influencing competition. Our work is closely related to Yildirim (2007). Similar to Yildirim, we adopt a generalized Tullock contest, but we introduce heterogeneous production technologies with fewer restrictions, as well as nonlinear effort cost functions. Yildirim conducts comparative statics of the prevailing voting rule for homogeneous agents and shows that a more inclusive voting rule—i.e., a larger minimum number of required votes—always decreases lower total effort. In contrast, we explore optimal rule design in a setting that allows for a general design objective, heterogeneous agents, and multiple design instruments (voting rule and recognition mechanism). Agents' heterogeneity catalyzes complex effects with varying voting rules or contest rules, which prevents standard comparative static analysis and differentiates our game from conventional bargaining or contest models.

Several papers examine the endogenous formation of a bargaining protocol without using a contest approach. Diermeier, Prato, and Vlaicu (2015, 2016) employ a pre-bargaining process to determine proposal power in bargaining over policy. In McKelvey and Riezman (1992, 1993); Muthoo and Shepsle (2014); and Eguia and Shepsle (2015), recognition probability is determined by seniority, which is endogenously voted on at the end of each session. Kim (2019) assumes that current and past proposers are excluded from the pool of eligible candidates when a round of bargaining fails to reach consensus. Jeon and Hwang (2022) assume that an agent's recognition probability and bargaining power depend on the previous bargaining outcome in a dynamic legislative bargaining model, which leads to an oligopolistic outcome as the result of an evolutionary process. Agranov, Cotton, and Tergiman (2020) examine, both theoretically and experimentally, a repeated multilateral bargaining model in which the agenda setter can retain his power with the majoritarian support of other committee members.

Our paper is closely related to Jeon and Hwang (2022) and Ali, Bernheim, and Fan (2019), who demonstrate that power concentration could arise in the equilibrium. The former study attributes the endogenous formation of oligopoly to the influence of past bargaining outcomes. The latter shows that the prevailing information structure could lead to extreme power in terms of distributive outcomes when the voting rule is not unanimous. Neither of them involves a contest of proposing rights or an endogenously set voting rule, which is the focus of this paper.

Our paper is also naturally linked to the literature on contest design and, particularly, that on optimally biased contests. We develop a technique similar to that of Fu and Wu (2020) and Fu, Wu, and Zhu (2023), who characterize the optimum without explicitly solving for the equilibrium. Our analysis complements these studies by embedding the contest in a multilateral sequential bargaining framework, which generates an endogenous prize spread.

The rest of the paper is structured as follows. Section 2 sets up the model and the design problem. Section 3 characterizes the equilibrium. Section 4 solves the optimal design problem and provides examples of the case with a single instrument, and Section 5 concludes. Proofs and derivations for examples are collected in Appendices A and B, respectively.

2 Model Setup

The game proceeds in two stages. In the first stage, a designer sets the rules that govern agents' subsequent interaction. In the second stage, a set of agents interact to divide a fixed sum of surplus, which is modeled as a multilateral sequential bargaining process with costly recognition, à la Yildirim (2007, 2010) and Ali (2015).

2.1 Multilateral Sequential Bargaining with Costly Recognition

A set of $n \geq 2$ agents, indexed by $\mathcal{N} := \{1, 2, \ldots, n\}$, decide how to divide a dollar. In each period $t = 0, 1, 2, \ldots$, one agent (proposer) makes a proposal $\boldsymbol{s}_t \in \triangle^{n-1} := \{(s_{1,t}, \ldots, s_{n,t}) : 0 \leq s_{i,t} \leq 1, \sum_{i \in \mathcal{N}} s_{i,t} = 1\}$, where $s_{i,t}$ denotes the share of the dollar each agent $i \in \mathcal{N}$ is to receive under this proposal. Agents simultaneously vote in favor of or against the proposal. We assume a "k-majority" voting rule—with $1 \leq k \leq n$—for this sequential bargaining process: The proposal is approved if at least k agents accept it (including the proposer). Specifically, $k = n$ implies a unanimous rule wherein the proposal can be vetoed by any single dissident; $k = \lfloor n/2 \rfloor + 1$ refers to a simple majority rule; with $k = 1$, the proposer dictates the decision process.

At the beginning of each period t, a contest takes place in which each agent exerts an effort $x_{i,t} \geq 0$ to vie for the proposing right, which incurs a cost $c_i(x_{i,t})$.

Proposer Recognition Mechanism For a given effort profile $\boldsymbol{x}_t := (x_{1,t}, \dots, x_{n,t})$, an agent i is recognized as the proposer for period t with a probability

$$p_i(\boldsymbol{x}_t) = \begin{cases} \dfrac{\tilde{f}_i(x_{i,t})}{\sum_{j \in \mathcal{N}} \tilde{f}_j(x_{j,t})}, & \sum_{j \in \mathcal{N}} \tilde{f}_j(x_{j,t}) > 0, \\ \dfrac{1}{n}, & \sum_{j \in \mathcal{N}} \tilde{f}_j(x_{j,t}) = 0, \end{cases} \tag{1}$$

where $\tilde{f}_i(\cdot)$ is called the impact function in the contest literature; it converts one's effort into his effective output in the competition, taking the form

$$\tilde{f}_i(\cdot) := \alpha_i f_i(\cdot) + \beta_i, \forall\, i \in \mathcal{N}. \tag{2}$$

The function $f_i(\cdot)$ describes agent i's actual production technology, while the multiplicative bias $\alpha_i \geq 0$ and additive headstart $\beta_i \geq 0$ are set by the designer as one part of the rules governing the contest for the proposing right. We will elaborate on the details later.

Preferences and Payoffs Each agent is risk neutral and has a discount factor $\delta_i \in (0, 1)$. Agents differ in the degrees of their patience. If a proposal is approved in period τ, an agent i's discounted payoff is

$$\Pi_i := \delta_i^\tau s_{i,\tau} - \sum_{t=0}^{\tau} \delta_i^t c_i(x_{i,t}),$$

where $s_{i,\tau}$ is the share he receives under the approved proposal and $c_i(x_{i,t})$ the effort cost incurred in each period $t \in \{0, \dots, \tau\}$.[3]

Solution Concept The bargaining game with costly recognition can be described as $\big\langle (\tilde{f}_i(\cdot))_{i \in \mathcal{N}}, (c_i(\cdot))_{i \in \mathcal{N}}, \boldsymbol{\delta}, k \big\rangle$, where $(\tilde{f}_i(\cdot))_{i \in \mathcal{N}}$ denotes the set of impact functions, $(c_i(\cdot))_{i \in \mathcal{N}}$ the set of effort cost functions, $\boldsymbol{\delta} := (\delta_1, \dots, \delta_n)$ the set of discounting factors, and k the voting rule.

We assume that agents use stationary strategies whereby for each period t, agents' period-t actions are independent of the history (see Theorem 1 for details of the strategies). We adopt the solution concept of the stationary subgame perfect equilibrium (SSPE) and drop

[3] If no agreement is reached, agent i's discounted payoff is $\Pi_i = -\sum_{t=0}^{+\infty} \delta_i^t c_i(x_{i,t})$.

the time subscript t throughout. A strategy profile is an SSPE if it is stationary and constitutes a subgame perfect equilibrium of the second-stage game.

We impose the following mild and standard regularity conditions to ensure equilibrium existence:

Assumption 1 *For each $i \in \mathcal{N}$, $f_i(\cdot)$ and $c_i(\cdot)$ are twice differentiable in $(0, +\infty)$, satisfying $f_i(0) = 0$, $f_i'(\cdot) > 0$, $f_i''(\cdot) \leq 0$, $c_i(0) = 0$, $c_i'(\cdot) > 0$, and $c_i''(\cdot) \geq 0$.*

2.2 Rule Design: Instruments and Objectives

We now lay out the design problem.

Design Instruments The designer adjusts two structural elements of the bargaining process. She sets the voting rule, which is implemented by choosing k, the minimum number of favorable votes required for the proposal's approval. Meanwhile, she can adjust the mechanism for proposer recognition (contest rules), which determines the probability of each agent's recognition for every given effort profile.

Recall that each agent i's impact function $\tilde{f}_i(\cdot)$ is given by (2). The designer imposes the multiplicative weights $\boldsymbol{\alpha} := (\alpha_1, \ldots, \alpha_n) \in \mathbb{R}^n_+ \setminus \{(0, \ldots, 0)\}$—which scale up or down one's output—and additive headstarts $\boldsymbol{\beta} := (\beta_1, \ldots, \beta_n) \in \mathbb{R}^n_+$. We can view $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ as nominal scoring rules. Alternatively, they can be viewed as the organizational resources assigned to agents that alter their productivity or influence (see, e.g., Fu and Wu, 2022).

Both multiplicative weights $\boldsymbol{\alpha}$ and additive headstarts $\boldsymbol{\beta}$ are broadly adopted in modeling biased contests: Epstein, Mealem, and Nitzan (2011) and Franke, Kanzow, Leininger, and Schwartz (2014), for instance, consider the former; Konrad (2002); Siegel (2009, 2014) and Kirkegaard (2012) focus on the latter; and Franke, Leininger, and Wasser (2018) and Fu and Wu (2020) allow for both. It is noteworthy that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ play different roles in impacting the contest's outcome: $\boldsymbol{\alpha}$ alter the marginal returns of agents' efforts, while $\boldsymbol{\beta}$ directly add to their effective output regardless of their efforts.

Design Objectives As will be shown later in Theorem 1, there is no delay in each SSPE, and thus agents exert effort at most once on the equilibrium path. The designer chooses $(\boldsymbol{\alpha}, \boldsymbol{\beta}, k)$ to maximize an objective function $\Lambda(\boldsymbol{x}, \boldsymbol{p})$, where $\boldsymbol{x} := (x_1, \ldots, x_n)$ and $\boldsymbol{p} := (p_1, \ldots, p_n)$ denote the profiles of equilibrium efforts and agents' recognition probabilities, respectively. The following regularity condition is imposed.

Assumption 2 *Fixing $\boldsymbol{p}$, $\Lambda(\boldsymbol{x}, \boldsymbol{p})$ weakly increases with x_i for each $i \in \mathcal{N}$.*

By Assumption 2, we focus on the scenario in which agents' efforts are productive and accrue to the designer's benefit. Consider, for example, executives who enhance their performance to climb the corporate ladder or party activists who contribute services to vie for leadership.

The objective function accommodates a diverse array of preferences. Consider, for example, $\Lambda(\boldsymbol{x}, \boldsymbol{p}) = \sum_{i\in\mathcal{N}} x_i - \lambda \sum_{i\in\mathcal{N}} \left|p_i - \frac{1}{n}\right|$, with $\lambda \geq 0$, which clearly satisfies Assumption 2. When $\lambda = 0$, this objective boils down to maximizing equilibrium total effort, which is conventionally assumed in the contest design literature. When $\lambda > 0$, the designer's payoff depends on the profile of agents' recognition probabilities. The term $\sum_{i\in\mathcal{N}} \left|p_i - \frac{1}{n}\right|$—i.e., the mean absolute deviation of $\boldsymbol{p}$—increases in the dispersion of $\boldsymbol{p}$. The function thus depicts a preference for a more equitable distribution of recognition opportunities, which compels the designer to set rules to reduce $\sum_{i\in\mathcal{N}} \left|p_i - \frac{1}{n}\right|$.[4]

Alternatively, consider $\Lambda(\boldsymbol{x}, \boldsymbol{p}) = \sum_{i\in\mathcal{N}} p_i x_i$, which is the expected winner's effort. Maximizing the expected winner's effort has gained increasing attention in the literature (e.g., Moldovanu and Sela, 2006; Barbieri and Serena, 2024). For instance, a firm often views its succession race as a process to develop managerial talent; the firm might benefit from the chosen successor's investment in their areas of expertise, since the losers often pursue alternative career paths, especially in high-profile public firms. For instance, James McNerney and Robert Nardelli joined 3M and Home Depot, respectively, after they lost the race to succeed Jack Welch at General Electric.

3 Equilibrium Existence and Characterization

We now characterize the equilibrium. Let $\boldsymbol{v} := (v_1, \ldots, v_n)$ be the set of agents' equilibrium expected payoffs and consider stage-undominated voting strategies, such that agents vote as if they were pivotal. Suppose that an agent is not recognized as the proposer. He accepts a proposal if his share exceeds the discounted continuation value—i.e., $s_i \geq \delta_i v_i$—and rejects it otherwise. The proposer, in contrast, needs to select $k-1$ agents to form the least costly winning coalition and offers them their continuation values. His expected vote-buying cost is

$$w_i = \sum_{j\neq i} \psi_{ij} \delta_j v_j,$$

[4]Eraslan and Merlo (2017) examine the distributive implications of voting rules. They show that unanimity may paradoxically lead to more unequal distributive outcome. It is noteworthy that in our context, the designer's fairness concern refers to her preference for ex ante distribution of bargaining power among agents—i.e., the recognition probability profile—instead of ex post distribution of the surplus.

where ψ_{ij} gives the probability of agent i's including j in his offer. For each $j \in \mathcal{N}$, we further define $\mu_j := \sum_{i \neq j} \psi_{ij} p_i$ as agent j's probability of being included in others' winning coalitions before a proposer is recognized.

For each agent $i \in \mathcal{N}$, the expected gross payoff conditional on being the proposer is $1 - w_i$ and that when not being selected is $\frac{\mu_i}{1-p_i}\delta_i v_i$. His effort x_i solves the maximization problem on the right-hand side of the following Bellman equation:

$$v_i = \max_{x_i \geq 0} \left\{ p_i(x_i, \boldsymbol{x}_{-i})(1 - w_i) + [1 - p_i(x_i, \boldsymbol{x}_{-i})] \times \frac{\mu_i}{1 - p_i(x_i, \boldsymbol{x}_{-i})} \delta_i v_i - c_i(x_i) \right\}, \quad (3)$$

which yields the following first-order condition:

$$\underbrace{c_i'(x_i)}_{\text{marginal cost of effort}} \geq \underbrace{\frac{\tilde{f}_i'(x_i)}{\tilde{f}_i(x_i)} \times p_i(1 - p_i) \times \overbrace{\left(1 - w_i - \frac{\mu_i}{1 - p_i}\delta_i v_i\right)}^{\text{effective prize spread}}}_{\text{marginal benefit of effort}}. \quad (4)$$

Equations (3) and (4) depict the strategic nature of this game. The term $1 - w_i - \frac{\mu_i}{1-p_i}\delta_i v_i$ is the payoff differential between winning the competition for recognition and losing it—i.e., the prize spread that motivates efforts. However, the prize spread is endogenously determined, since w_i, p_i, μ_i, and v_i all depend on agents' effort profile $\boldsymbol{x} = (x_1, \ldots, x_n)$ and vice versa. These nuances differentiate the model from a standard contest with a fixed prize or a standard multilateral sequential bargaining game, dismissing the regularity typically assumed in conventional frameworks. Our analysis obtains the following.

Theorem 1 *Suppose that Assumption 1 holds. For each game $\langle (\tilde{f}_i(\cdot))_{i\in\mathcal{N}}, (c_i(\cdot))_{i\in\mathcal{N}}, \boldsymbol{\delta}, k\rangle$, there exists an SSPE characterized by $(\boldsymbol{x}, \boldsymbol{v})$ and $\{\psi_{ij}\}_{i\neq j}$. In the equilibrium, each agent $i \in \mathcal{N}$ exerts effort x_i in each period. If selected as the proposer, he forms a winning coalition of $k - 1$ agents such that agent j is included with probability ψ_{ij} and offers the agent $\delta_j v_j$. Otherwise, he accepts a proposer's offer if and only if his share is no less than $\delta_i v_i$. The equilibrium is unique when $k = 1$.*

Theorem 1 establishes equilibrium existence of the game, which paves the way for optimal rule design. The setting of Yildirim (2007) assumes $\tilde{f}_i(0) = 0$, linear cost function, and weakly decreasing elasticity $x_i\tilde{f}_i'(x_i)/\tilde{f}_i(x_i)$ for each $i \in \mathcal{N}$. We relax these restrictions, allowing for headstarts β_i—which could lead to $\tilde{f}_i(0) \neq 0$—nonlinear cost functions $c_i(\cdot)$, and unrestricted elasticity conditions.

The equilibria might be nonunique, and a closed-form equilibrium solution is in general unavailable in our context.[5] It is well known that asymmetric contests cannot be solved in closed form when the number of contestants exceeds two. The nuances caused by the endogenous payoff structure entails further complications. This nullifies the usual implicit programming approach to optimal design. We develop a technique in line with Fu and Wu (2020), which enables us to characterize the optimum without explicitly solving for the equilibrium.

4 Optimal Design of Organizational Rules

We now explore the optimal organizational rules. We first examine the respective nature of each set of structural elements—i.e., voting rule k and recognition mechanism $(\boldsymbol{\alpha}, \boldsymbol{\beta})$—and their respective roles in shaping agents' incentives and behavior. We first demonstrate how the implications differ from the conventional wisdom in the literature. We then present the main results—i.e., the optimum when the designer has full flexibility to adjust $(\boldsymbol{\alpha}, \boldsymbol{\beta}, k)$.

4.1 Role of the Voting Rule

We begin with the voting rule k. Suppose that the bargaining process implements a more inclusive voting rule, i.e., increasing k. It generates an effect on agents' prize spreads, which we call the (direct) *prize effect*. A larger k changes both his winning prize—i.e., $1 - w_i$—and losing prize, i.e., $\frac{\mu_i}{1-p_i}\delta_i v_i$. A proposer has to buy more votes if he wins, and he needs to buy votes from a different set of his peers; each peer would demand a different offer, since their continuation values change: All of these change w_i. Further, a losing candidate may expect a different payoff because he is more likely to be included in some winning coalitions, while the minimum share he would accept also varies with the change in his continuation value: These change $\frac{\mu_i}{1-p_i}\delta_i v_i$ accordingly.

Overall, the effect is ambiguous. More importantly, this effect is *nonuniform* among asymmetric agents, which further leads to the (indirect) *rebalancing effect*. Imagine $k = 1$, such that all agents have a prize spread of 1 irrespective of their patience levels, since the proposer can expropriate all surplus without rallying support from his peers and one ends up with nothing once he fails to be recognized. Suppose that k increases to 2. Agents' patience δ_i now plays a role in determining prize spreads. Ceteris paribus, the most patient agent is least likely to be included in a winning coalition. His losing value, $\frac{\mu_i}{1-p_i}\delta_i v_i$, tends

[5] Fixing a recognition probability profile—i.e., fixing an effort profile—the literature on multilateral bargaining has noticed that there exist multiple equilibria that differ in $\{\psi_{ij}\}$, but they result in the same profile of $(\mu_1, \ldots, \mu_n)$. An additional layer of equilibrium multiplicity may arise within our context in the sense that agents' effort profile may differ across equilibria.

to rise less than the others, causing a smaller decrease in his prize spread than those of the others. The nonuniform changes in agents' prize spreads tilt the playing field of the contest, which, together with the heterogeneity in impact and cost functions, alter agents' incentives indefinitely.

We construct an example to illustrate the subtlety. Assume that the designer aims to maximize the total effort, i.e., with an objective function $\Lambda(\boldsymbol{x}, \boldsymbol{p}) = \sum_{i \in \mathcal{N}} x_i$. Fixing a neutral recognition mechanism $\boldsymbol{\alpha} = (1, \ldots, 1)$ and $\boldsymbol{\beta} = (0, \ldots, 0)$, we explore the optimal voting rule.

Example 1 *Suppose that* $n = 4$, $f_i(x_i) = \eta_i x_i$, $c_i(x_i) = \eta_i x_i$, *with* $\boldsymbol{\eta} = (1, 0.2, 0.2, 0.2)$ *and* $\boldsymbol{\delta} = (0.1, 0.5, 0.5, 0.5)$. *The recognition mechanism is fixed and required to be neutral, with* $\boldsymbol{\alpha} = (1, 1, 1, 1)$ *and* $\boldsymbol{\beta} = (0, 0, 0, 0)$. *The designer chooses* $k \in \{1, 2, 3, 4\}$ *to maximize the total effort. The equilibria under different voting rules are depicted in Table 1, which demonstrates that the total effort of the process is maximized by setting* $k = 2$.

	$k=1$	$k=2$	$k=3$	$k=4$
Winning probability of agent 1	0.2500	0.2322	0.2421	0.2500
Winning probability of agents 2-4	0.2500	0.2559	0.2526	0.2500
Equilibrium effort of agent 1	0.1875	0.1711	0.1656	0.1570
Equilibrium efforts of agents 2-4	0.9375	0.9433	0.8641	0.7849
Total effort	3.0000	3.0011	2.7578	2.5116

Table 1: Equilibrium Outcomes in Example 1.

The results are summarized in Table 1. There are two types of agents: 1 impatient agent and 3 patient agents. When $k = 1$, heterogeneity in effort cost and that in impact function perfectly offset each other and all agents win with equal probability. Each agent needs to pay the vote-buying cost when k increases to 2 and thus the prize effect arises, which tends to reduce the prize spread and the equilibrium effort. The three patient agents have higher levels of patience and are less likely to be included in other agents' winning coalition, since high patience elevates their continuation value and therefore others' costs of buying their votes. Conversely, the impatient agent is always included in the winning coalition, which increases his losing value. As a result, patient agents have a larger prize spread and therefore a stronger prize incentive. The nonuniform changes in prize spreads alter the balance of the playing field and lead to the rebalancing effect, which increases patient agents' winning probability, intensifies their competition, and tends to increase their equilibrium effort. In this example, the rebalancing effect dominates the opposing prize effect for the three patient agents. Fixing a neutral recognition mechanism, the total effort is nonmonotone in k, being maximized by $k = 2$.

This observation sharply contrasts with the result of Yildirim (2007). With symmetric agents, Yildirim shows that total effort strictly decreases with k. Intuitively, with symmetric agents, an increase in k decreases agents' prize spreads and weakens their incentives, while the rebalancing effect is entirely muted due to symmetry. As shown in Table 1, for fixed $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, the value of the objective function is nonmonotone with respect to k and no explicit prediction can be obtained in general in the case in which agents are heterogeneous.

4.2 Role of the Recognition Mechanism

We now examine the role played by the recognition mechanism. The designer sets $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, while fixing the voting rule k, to maximize the objective function

$$\Lambda(\boldsymbol{x}, \boldsymbol{p}) = \sum_{i \in \mathcal{N}} x_i - \lambda \sum_{i \in \mathcal{N}} \left| p_i - \frac{1}{n} \right|, \text{ with } \lambda > 0. \tag{5}$$

Example 2 *Suppose that* $n = 3$*,* $k = 2$*,* $f_i(x_i) = x_i$*, and* $c_i(x_i) = c_i x_i$ *with* $(c_1, c_2, c_3) = (1, 1, c)$*. Let* $(\delta_1, \delta_2, \delta_3) = (\frac{3}{8}, \frac{1}{2}, \frac{12}{13})$*. Assume that* λ *is sufficiently large and* c *is sufficiently small, with* $\lambda \gg \frac{1}{c} \gg 1$*.*

The objective function can be maximized by a recognition mechanism with $\boldsymbol{\alpha}^* = (\frac{62Y}{35}, \frac{62Y}{37}, \frac{62Yc}{39})$ *and* $\boldsymbol{\beta}^* = (0, \frac{17Y}{222}, 0)$*, where* $Y > 0$ *is an arbitrary positive constant. The game yields an equilibrium outcome of* $\boldsymbol{x} = (\frac{70}{372}, \frac{57}{372}, \frac{78}{372c})$ *and* $\boldsymbol{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$*. The designer's payoff is* $\Lambda = \frac{127}{372} + \frac{78}{372c}$*.*

	Agent 1	Agent 2	Agent 3
Equilibrium efforts	70/372	57/372	$78/(372c)$
Winning probability	1/3	1/3	1/3
Equilibrium payoff	56/372	72/372	39/372
Winning coalition	$\{1, 2\}$	$\{1, 2\}$	$\{1, 3\}$

Table 2: Equilibrium Outcomes in Example 2.

Notably, the designer awards a positive headstart to agent 2. This stands in contrast to findings in the literature on contest design. Fu and Wu (2020), for instance, formally establish the suboptimality of a headstart, and show that adjusting $\boldsymbol{\alpha}$ suffices to achieve the optimum. The contrast unveils how the endogenous prize structure differentiates the game from a standard static contest.

With $c_1 = c_2 > c_3$ and $\delta_1 < \delta_2 < \delta_3$, agent 3 is ex ante the strongest contender, followed by agent 2, then agent 1. The designer would benefit if agent 3 can be sufficiently incentivized given his low effort cost, which requires a larger prize spread for the agent. For this purpose, the designer can seek to reduce agent 1's continuation value, which decreases agent 3's vote-buying cost—i.e., w_3—given that by Table 2, agent 3 would include agent 1 in his winning coalition.

Further, by Table 2, agent 1 would buy agent 2's vote upon being the proposer. The designer can increase agent 2's continuation value to render agent 1 worse off, which can be achieved by awarding agent 2 either a headstart $\beta_2 > 0$ or a larger α_2. The former is more effective in this context: Both increase agent 2's recognition probabilities and improve his payoffs. However, a larger α_2 increases the marginal benefit of effort, which promotes his effort supply; effort is costly and, in turn, reduces agent 2's payoff, (partially) offsetting the payoff-improving effect of a larger α_2.

These subtleties, as the artifact of the dynamic bargaining process, are absent in a simple static contest. The prize is fixed in a standard contest, so the contest rule $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ only rebalances the playing field and does not alter agents' prize incentives. Multiplicative biases $\boldsymbol{\alpha}$ can more effectively motivate efforts due to their direct impact on the marginal benefits of efforts, rendering headstarts $\boldsymbol{\beta}$ redundant. In contrast, the prize spreads in our context, when $k \geq 2$, endogenously depend on agents' equilibrium efforts; so a change in the contest rule catalyzes not only a (direct) rebalancing effect but also an (indirect) prize effect. In this particular example, varying $\boldsymbol{\beta}$ creates an opportunity to exploit the endogenous payoff structure of the game.

In summary, due to the complexity, no general comparative statics can be obtained with respect to the recognition mechanism $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ under fixed k.

4.3 Main Result

We now explore the general optimization problem that allows the designer to set k and $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ altogether to maximize the objective function $\Lambda(\boldsymbol{x}, \boldsymbol{p})$. Despite the complexity that arises when either k or $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ varies alone, an optimum exists with unambiguous implications. Our analysis concludes the following.

Theorem 2 *Suppose that Assumptions 1 and 2 hold. When the designer can flexibly choose $(\boldsymbol{\alpha}, \boldsymbol{\beta}, k)$, the optimum involves a dictatorial voting rule ($k = 1$) and zero headstart ($\boldsymbol{\beta} = \mathbf{0}$).*

By Theorem 2, a dictatorial voting rule always emerges in the optimum, although the specific form of the associated recognition mechanism $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ depends on the particular context. The proposer does not need a winning coalition and relinquish his share. As a result, each agent has a fixed prize spread of 1 in the optimum, and their patience levels do not affect an equilibrium outcome. It is noteworthy that the optimum calls for ex post concentration of bargaining power, even if she cares about an even distribution of recognition opportunity ex ante.

The logic of these results can be interpreted in light of the interactions between the prize and the rebalancing effects postulated earlier. A dictatorial voting rule ($k = 1$) generates a maximized prize spread, since both w_i and $\frac{\mu_i}{1-p_i}\delta_i v_i$ are zero. This provides the largest prize incentive to agents and tempts them to strive for recognition. The designer can then adjust the recognition mechanism $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ to optimally tilt the playing field if necessary. It is noteworthy that the ambiguous indirect prize effect caused by a change in $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is entirely muted because the prize spread is fixed when $k = 1$ and does not depend on agents' continuation values. The findings from the contest literature (Fu and Wu, 2020) can be reinstated: The designer can induce any profile of equilibrium winning odds by adjusting $\boldsymbol{\alpha}$, and additive headstarts $\boldsymbol{\beta}$ are redundant.

In summary, the joint design achieves the optimum, and the two sets of instruments play distinct roles: The voting rule—with $k = 1$—generates the maximum and a fixed prize spread, while the multiplicative biases $\boldsymbol{\alpha}$ optimally exploit agents' heterogeneity in terms of their innate abilities and sets the optimal competitive balance.

4.4 Extensions, Discussions, and Implications

In what follows, we first present further discussions that explore the limit of our analysis, then elaborate on the implications of our results for organizational design.

4.4.1 Further Analysis

Theorem 2 establishes the general optimality of a dictatorial voting rule, although the specific form of the associated recognition mechanism depends on the particular environment. This observation naturally inspires the following question regarding the general effect of varying k: Despite the unavailability of the comparative statics of k when the voting rule changes alone, as shown in Example 1, does a less inclusive voting rule—i.e., a smaller k—necessarily improve the value of the objective function (5) when the designer can adjust the recognition mechanism optimally for every given k?

The following example demonstrates that this conjecture does not hold in general.

Example 3 *Suppose $n = 7$ and $\delta_i = 0.999$ for all agents $i \in \mathcal{N}$. Each agent has a production technology $f_i(x_i) = x_i$. We construct the following vectors: $\tilde{\boldsymbol{p}} := (0.005, 0.005, 0.005, 0.1, 0.1, 0.1, 0.685)$, $r = 839.9 \times \tilde{p}_7(1 - \tilde{p}_7)$, and $\tilde{\boldsymbol{x}} = (0.0037, 0.0037, 0.0037, 0.0144, 0.0144, 0.0144,$*

$0.0001^{\frac{1}{r}}$). *Further, agents' effort cost functions take the following form:*

$$c_i(x_i) = \begin{cases} x_i, & x_i \leq \tilde{x}_i \text{ and } i \leq 6, \\ x_i^r, & x_i \leq \tilde{x}_i \text{ and } i = 7, \\ \tilde{x}_i + \gamma(x_i - \tilde{x}_i), & x_i > \tilde{x}_i \text{ and } i \leq 6, \\ \tilde{x}_i^r + \gamma(x_i - \tilde{x}_i), & x_i > \tilde{x}_i \text{ and } i = 7, \end{cases}$$

where γ is a sufficiently large constant. Assume an objective function (5)*—i.e., $\Lambda = \sum_{i\in\mathcal{N}} x_i - \lambda \sum_{i\in\mathcal{N}} |p_i - \tilde{p}_i|$—with a sufficiently large λ. The designer can freely set $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. It can be verified that setting k to either* 5 *or* 1 *maximizes the objective function, while $k = 4$ is suboptimal, which indicates the nonmonotonicity of the designer's payoff with respect to k.*

Example 3 sheds light on the nuances of this game. On the one hand, increasing k requires a proposer to buy more votes, which, ceteris paribus, directly reduces each agent's winning prize and effort incentive. On the other hand, the cost for each vote may also change due to the altered dynamics involved in the bargaining process; this indirectly affects agents' winning prizes and could either increase or decrease them. By Example 3, the cost of an individual agent's vote can be reduced when the voting rule becomes more inclusive; so the latter indirect effect may dominate the former direct effect, which results in the nonmonotonicity of the designer's payoff with respect to k.

Our analysis further obtains the following.

Theorem 3 *Suppose that $\delta_i \leq \frac{1}{2}$ for each $i \in \mathcal{N}$ and Assumptions 1 and 2 hold. If the designer can flexibly adjust $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, the objective function Λ is weakly decreasing in k.*

Theorem 3 establishes the general monotonicity of k with additional restrictions on δ_i. That is, the designer prefers a less inclusive voting rule when agents are relatively impatient—i.e., when δ_i is bounded from above by $1/2$. The result is intuitive. When agents are relatively impatient, the abovementioned nuanced indirect effect triggered by changes in k and $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ are limited. The direct effect of a rising k—i.e., elevating vote-buying cost and reducing prize spread—dominates the trade-off and leads to our prediction.

4.4.2 Implications

Our analysis provides valuable insights for designing organizational rules that guide the internal processes of distributing power and allocating resources. For instance, our results could inform renewed debates on intra-party democracy (IPD)—i.e., a party's practice of selecting leaders, making key decisions, and deploying resources (Cross, 2013; Poguntke et al., 2016). Strong advocacy exists for reforms to democratize the internal structures of political parties and address a wide array of concerns, such as restoring public trust, promoting inclusion and openness, and rebuilding the democratic link between citizens and governments (Dalton, Farrell, and McAllister, 2011; Scarrow, 1999, 2014). As documented by Cross (2013), parties' practices differ substantially in reality. For example, party leaders in the UK and Belgium were often chosen by party elites through acclamation, even in leadership contests with broad selectorates or full member votes, whereas Canada sets a low threshold for access to the contest by putting candidates on an equal footing. Parties also vary significantly in terms of the inclusiveness of their decision-making procedures and the authority granted to leaders (Poguntke et al., 2016).

The practice of IPD could be a double-edged sword, yielding both positive and negative effects, especially given the external accountability pressure imposed by electoral competition (Cross and Katz, 2013). A wealth of scholarly effort has been dedicated to the debate on IPD in political science literature, primarily addressing its normative concerns. The economics literature, however, remains relatively silent in terms of providing formal analysis regarding the ramifications of power distribution or internal party organization. Dewan and Squintani (2016) provide a rationale for faction formation within a party, and demonstrate that factionalism facilitates information aggregation and empowers moderate politicians. Caillaud and Tirole (2002) demonstrate how competition between factions for candidate selection can be managed for electoral success. Crutzen, Castanheira, and Sahuguet (2010) portray a context in which two parties choose their own internal structures—i.e., the competitiveness of candidate selection mechanisms—and identify critical effects of organizational formats on electoral competition. Our paper focuses on a different context and a different set of instruments to streamline the internal distributive process, and thereby complements this strand of the literature.

As another example, our analysis could yield insights for the burgeoning experimentation with blockchain-based decentralized autonomous organizations (DAOs). A DAO is an organization in which members propose projects and vote to decide whether a project can be funded by community resources. Although the execution of organizational decisions is automated through smart contracts based on blockchain-enabled computer programs, the prescribed procedures (voting and funding) are subject to the choice of the DAO's initiating institution.[6]

The governance of DAOs has sparked extensive debates on both equity and efficiency grounds (Zhao, Ai, Lai, Luo, and Benitez, 2022). The organizational rules of a DAO typically

[6] Readers are referred to https://en.wikipedia.org/wiki/Decentralized_autonomous_organization#cite_note-17.

mainly consist of two elements: (i) the threshold for passing a proposal—e.g., the number of favorable votes required and/or the length of the voting window—and (ii) the mechanism for filtering or selecting proposals for voting. This scenario provides a particularly relevant context for our analysis. According to our results, the optimum reduces the process to a proposal selection mechanism while setting a minimum threshold—i.e., leaving decision rights exclusively to the proposer. This holds even if the design objective involves concerns about the even distribution of decision power ex ante. Although operating a DAO and implementing its rules could present various other challenges beyond the scope of our model, the analysis offers a useful perspective.

5 Concluding Remarks

In this paper, we model organizational politics as a sequential multilateral bargaining game with costly recognition, in which a proposer suggests a plan to divide a fixed amount of resources and the proposer is determined through a contest. We explore the optimal organizational rules, with a designer who deploys two sets of design instruments: (i) the voting rule that governs how proposals are accepted or rejected and (ii) the recognition mechanism that determines how the proposer is selected based on agents' productive efforts. We demonstrate that when the designer can choose both structural elements, the optimum always involves a dictatorial rule that maximizes a general objective function. This simplifies the bargaining game with costly recognition and condenses it into a standard contest.

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Appendix A: Proofs

Proof of Theorem 1

Proof. We first characterize the SSPE assuming its existence, then prove equilibrium existence.

Equilibrium Characterization Denote by V^Δ the k-th lowest continuation value. Let $\mathcal{N}_1 := \{i \in \mathcal{N} : \delta_i v_i < V^\Delta\}$, $\mathcal{N}_2 := \{i \in \mathcal{N} : \delta_i v_i = V^\Delta\}$, and $\mathcal{N}_3 := \{i \in \mathcal{N} : \delta_i v_i > V^\Delta\}$. Evidently, agent i, when becoming the proposer, buys out the votes of the cheapest "winning coalition"—i.e., $\mathcal{N}_1$ and a subset of $\mathcal{N}_2$, from which we can conclude

$$\psi_{ij} \begin{cases} = 1, & j \in \mathcal{N}_1, \\ \in [0,1], & j \in \mathcal{N}_2, \\ 0, & j \in \mathcal{N}_3, \end{cases} \text{ and } \mu_i \begin{cases} = 1 - p_i, & i \in \mathcal{N}_1, \\ \in [0, 1-p_i], & i \in \mathcal{N}_2, \\ = 0, & i \in \mathcal{N}_3. \end{cases} \tag{6}$$

Define

$$V_L := 1 - \sum_{j \in \mathcal{N}_1} \delta_j v_j - \left(k - |\mathcal{N}_1|\right) V^\Delta. \tag{7}$$

Agent i's expected cost is then

$$w_i = \begin{cases} 1 - V_L - \delta_i v_i, & i \in \mathcal{N}_1, \\ 1 - V_L - V^\Delta, & \text{otherwise.} \end{cases}$$

The effective prize spread $1 - w_i - \frac{\mu_i}{1-p_i}\delta_i v_i$ in (4) can be expressed as

$$1 - w_i - \frac{\mu_i}{1-p_i}\delta_i v_i = V_L + \frac{1-\mu_i-p_i}{1-p_i}V^\Delta = \begin{cases} V_L, & i \in \mathcal{N}_1, \\ V_L + \frac{1-p_i-\mu_i}{1-p_i}V^\Delta, & i \in \mathcal{N}_2, \\ V_L + V^\Delta, & i \in \mathcal{N}_3. \end{cases} \tag{8}$$

We are ready to lay out the conditions for equilibrium characterization. An SSPE can be characterized by $(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{p}, \boldsymbol{\mu}, V_L, V^\Delta)$. Combining (4) and (8) yields

$$\frac{c_i'(x_i)\tilde{f}_i(x_i)}{\tilde{f}_i'(x_i)} \geq p_i(1-p_i)\left(V_L + \frac{(1-p_i-\mu_i)V^\Delta}{1-p_i}\right). \tag{9}$$

Next, consider the expected payoff v_i. By (3), we have

$$v_i = p_i(1-w_i) + \mu_i\delta_i v_i - c_i(x_i) = \begin{cases} \frac{1}{1-\delta_i}\left(p_i V_L - c_i(x_i)\right), & i \in \mathcal{N}_1, \\ \frac{V^\Delta}{\delta_i}, & i \in \mathcal{N}_2, \\ p_i(V_L + V^\Delta) - c_i(x_i), & i \in \mathcal{N}_3. \end{cases} \tag{10}$$

Combining (3), (6), and (10) yields

$$\mu_i \begin{cases} = 1 - p_i, & i \in \mathcal{N}_1, \\ \in [0, 1-p_i] \text{ solves } \frac{V^\Delta}{\delta_i} = p_i V_L + (\mu_i + p_i)V^\Delta - c_i(x_i), & i \in \mathcal{N}_2, \\ = 0, & i \in \mathcal{N}_3. \end{cases} \tag{11}$$

Each agent chooses exactly $k-1$ agents in his winning coalition—i.e., $\sum_{j \neq i} \psi_{ij} = k-1$, $\forall i \in \mathcal{N}$. Therefore,

$$\sum_{i \in \mathcal{N}} \mu_i = \sum_{i \in \mathcal{N}} \sum_{j \neq i} \psi_{ji} p_j = \sum_{j \in \mathcal{N}} p_j \sum_{i \neq j} \psi_{ji} = \sum_{j \in \mathcal{N}} (k-1)p_j = k-1. \tag{12}$$

Last, (7) can be rewritten as

$$V_L + \sum_{i \in \mathcal{N}_1} (\delta_i v_i) + \left(k - |\mathcal{N}_1|\right) V^\Delta = 1. \tag{13}$$

To characterize an SSPE, it suffices to find $(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{p}, \boldsymbol{\mu}, V^\Delta, V_L)$ that satisfies (9)-(13).

Equilibrium Existence Let $Y := \sum_{i \in \mathcal{N}} \tilde{f}_i(x_i)$. By (1), we have $p_i = \tilde{f}_i(x_i)/Y$, which implies that

$$x_i = \tilde{f}_i^{-1}(Y p_i), \text{ for } p_i \in \left[\tilde{f}_i(0)/Y, 1\right], \tag{14}$$

and

$$\sum_{i \in \mathcal{N}} p_i = 1. \tag{15}$$

Substituting (14) into (9) yields

$$\frac{Y c_i'\left(\tilde{f}_i^{-1}(Y p_i)\right)}{\tilde{f}_i'\left(\tilde{f}_i^{-1}(Y p_i)\right)} \geq (1-p_i)(V_L + V^\Delta) - \mu_i V^\Delta, \text{with equality holding if } p_i > \frac{\tilde{f}_i(0)}{Y}. \tag{16}$$

Rewriting (11) and (12) and substituting (10) into (13) yield

$$\mu_i = \frac{1}{V^\Delta}\text{med}\left\{0, V^\Delta(1-p_i), \frac{V^\Delta}{\delta_i} - p_i(V_L+V^\Delta) + c_i\left(\tilde{f}_i^{-1}(Yp_i)\right)\right\}, \tag{17}$$

$$\sum_{i\in\mathcal{N}} \mu_i = k-1, \tag{18}$$

and

$$\sum_{i\in\mathcal{N}_1} \frac{\delta_i}{1-\delta_i}\left[p_iV_L - c_i\left(\tilde{f}_i^{-1}(Yp_i)\right)\right] + \left(k-|\mathcal{N}_1|\right)V^\Delta + V_L = 1, \tag{19}$$

where med$\{\cdot,\cdot,\cdot\}$ gives the median of the input.

To prove equilibrium existence, it suffices to show that there exists $(\boldsymbol{p}, \boldsymbol{\mu}, Y, V^\Delta, V_L)$ to satisfy conditions (15)-(19). The proof consists of four steps. First, fixing (Y, V^Δ, V_L), we show that there exists a unique $(\boldsymbol{p}, \boldsymbol{\mu})$ to satisfy (16) and (17). Second, fixing (V^Δ, V_L), there exists $Y \geq \sum_{i\in\mathcal{N}} \tilde{f}_i(0)$ to satisfy (15). Third, fixing V_L, there exists V^Δ to satisfy (18). Last, we show that there exists V_L to satisfy (19).

Step I Substituting (17) into (16) yields

$$\frac{Yc_i'\big(\tilde{f}_i^{-1}(Yp_i)\big)}{\tilde{f}_i'\big(\tilde{f}_i^{-1}(Yp_i)\big)} \geq \text{med}\left\{(1-p_i)(V_L+V^\Delta), (1-p_i)V_L, V_L+V^\Delta - \frac{V^\Delta}{\delta_i} - c_i\big(\tilde{f}_i^{-1}\big(Yp_i\big)\big)\right\}, \tag{20}$$

with equality holding if $p_i > \frac{\tilde{f}_i(0)}{Y}$.

Let

$$\phi(p_i) := \frac{Yc_i'\big(\tilde{f}_i^{-1}(Yp_i)\big)}{\tilde{f}_i'(\tilde{f}_i^{-1}(Yp_i))} - \text{med}\left\{(1-p_i)(V_L+V^\Delta), (1-p_i)V_L, V_L+V^\Delta - \frac{V^\Delta}{\delta_i} - c_i\big(\tilde{f}_i^{-1}(Yp_i)\big)\right\}.$$

By Assumption 1 and (2), $\tilde{f}_i(\cdot)$ is increasing and concave, which implies that $\phi(\cdot)$ strictly increases with p_i. Therefore, if $\phi\big(\frac{\tilde{f}_i(0)}{Y}\big) \geq 0$, or equivalently, if

$$\frac{Yc_i'(0)}{\tilde{f}_i'(0)} \geq \text{med}\left\{\left(1-\frac{\tilde{f}_i(0)}{Y}\right)V_L, \left(1-\frac{\tilde{f}_i(0)}{Y}\right)(V_L+V^\Delta), V_L+V^\Delta - \frac{V^\Delta}{\delta_i}\right\}, \tag{21}$$

then $p_i = \frac{\tilde{f}_i(0)}{Y}$. Otherwise, if $\phi\big(\frac{\tilde{f}_i(0)}{Y}\big) < 0$, or equivalently, if

$$\frac{Y c_i'(0)}{\tilde{f}_i'(0)} < \text{med}\left\{\left(1 - \frac{\tilde{f}_i(0)}{Y}\right) V_L, \left(1 - \frac{\tilde{f}_i(0)}{Y}\right)(V_L + V^\Delta), V_L + V^\Delta - \frac{V^\Delta}{\delta_i}\right\}, \tag{22}$$

then $p_i > \frac{\tilde{f}_i(0)}{Y}$; moreover, p_i is uniquely pinned down by $\phi(p_i) = 0$, or equivalently,

$$\frac{Y c_i'\big(\tilde{f}_i^{-1}(Y p_i)\big)}{\tilde{f}_i'(\tilde{f}_i^{-1}\big(Y p_i)\big)} = \text{med}\left\{(1 - p_i)(V_L + V^\Delta), (1 - p_i)V_L, V_L + V^\Delta - \frac{V^\Delta}{\delta_i} - c_i\big(\tilde{f}_i^{-1}(Y p_i)\big)\right\}. \tag{23}$$

Further, μ_i can be uniquely solved from (17). Therefore, fixing (Y, V^Δ, V_L), there exists a unique pair (p_i, μ_i) to satisfy (16) and (17), which we denote by $\big(p_i(Y, V^\Delta, V_L), \mu_i(Y, V^\Delta, V_L)\big)$ with slight abuse of notation.

Step II We show that fixing (V^Δ, V_L) and $\big\{p_i(Y, V^\Delta, V_L), \mu_i(Y, V^\Delta, V_L)\big)\}_{i\in\mathcal{N}}$, there exists $Y \geq \sum_{i\in\mathcal{N}} \tilde{f}_i(0)$ to satisfy (15). By definition of $p_i(Y, V^\Delta, V_L)$, we have that $p_i(Y, V^\Delta, V_L) \geq \frac{\tilde{f}_i(0)}{Y}$, which implies

$$\sum_{i\in\mathcal{N}} p_i\left(\sum\nolimits_{j\in\mathcal{N}} \tilde{f}_j(0), V^\Delta, V_L\right) \geq 1.$$

Next, we claim that

$$\lim_{Y\to+\infty} \sum_{i\in\mathcal{N}} p_i(Y, V^\Delta, V_L) = 0.$$

To see this, first consider the case of $\tilde{f}_i'(0) < +\infty$. Then (21) holds as Y approaches infinity, in which case $p_i = \frac{\tilde{f}_i(0)}{Y}$ and

$$\lim_{Y\to+\infty} p_i(Y, V^\Delta, V_L) = \lim_{Y\to+\infty} \frac{\tilde{f}_i(0)}{Y} = 0.$$

Next, consider the case of $\tilde{f}_i'(0) = +\infty$. Then (22) holds for all Y and $p_i(Y, V^\Delta, V_L)$ solves (23). As Y approaches infinity, the right-hand side of (23) approaches infinity; therefore, the left-hand side must be finite, indicating that $p_i(Y, V^\Delta, V_L)$ approaches 0.

By the intermediate value theorem, there exists $Y \geq \sum_{i\in\mathcal{N}} \tilde{f}_i(0)$ such that

$$\sum_{i\in\mathcal{N}} p_i(Y, V^\Delta, V_L) = 1.$$

Fixing (V^Δ, V_L), we denote the largest Y that solves the above equation by $Y(V^\Delta, V_L)$ in the subsequent analysis.

Step III Fixing V_L, $Y(V^\Delta, V_L)$, and $\{p_i(Y, V^\Delta, V_L), \mu_i(Y, V^\Delta, V_L))\}_{i\in\mathcal{N}}$, we show that there exists V^Δ such that (18) holds, i.e.,

$$\sum_{i\in\mathcal{N}} \mu_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right) = k - 1. \tag{24}$$

First, consider the case in which V^Δ approaches 0. For each $i \in \mathcal{N}$, when $p_i = \frac{\tilde{f}_i(0)}{Y}$, we have that

$$\begin{aligned}
&\lim_{V^\Delta \searrow 0} \mu_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right) \\
=&\lim_{V^\Delta \searrow 0} \frac{1}{V^\Delta} \text{med} \left\{0, V^\Delta \left(1 - \frac{\tilde{f}_i(0)}{Y(V^\Delta, V_L)}\right), \frac{V^\Delta}{\delta_i} - \frac{\tilde{f}_i(0)}{Y(V^\Delta, V_L)}(V_L + V^\Delta)\right\} = 0,
\end{aligned}$$

where the second equality follows from the fact that $\frac{V^\Delta}{\delta_i} - \frac{\tilde{f}_i(0)}{Y(V^\Delta,V_L)}(V_L + V^\Delta) \leq 0 \leq V^\Delta\big(1 - \frac{\tilde{f}_i(0)}{Y(V^\Delta,V_L)}\big)$ as V^Δ approaches 0.

When $p_i > \frac{\tilde{f}_i(0)}{Y}$, by (17), $\mu_i\left(Y(V^\Delta, V_L), V^\Delta, V_L\right) = 0$ for sufficiently small V^Δ. Therefore, we have that

$$\lim_{V^\Delta \searrow 0} \sum_{i\in\mathcal{N}} \mu_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right) = 0.$$

Next, consider the case in which V^Δ approaches infinity. For each $i \in \mathcal{N}$, we have that

$$\begin{aligned}
0 \leq& V^\Delta \left[1 - p_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right)\right] \\
\leq& \frac{V^\Delta}{\delta_i} - p_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right)(V_L + V^\Delta) + c_i \left(\tilde{f}_i^{-1}\left(Y(V^\Delta, V_L) p_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right)\right)\right);
\end{aligned}$$

together with (17), we can obtain that

$$\mu_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right) = 1 - p_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right), \text{ as } V^\Delta \to +\infty.$$

Therefore, we have that

$$\lim_{V^\Delta \to +\infty} \sum_{i\in\mathcal{N}} \mu_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right) = \lim_{V^\Delta \to +\infty} \sum_{i\in\mathcal{N}} \left[1 - p_i \left(Y(V^\Delta, V_L), V^\Delta, V_L\right)\right] = n - 1.$$

Note that $\mu_i(Y, V^\Delta, V_L)$ and $Y(V^\Delta, V_L)$ are continuous for all $i \in \mathcal{N}$, and $0 \leq k - 1 \leq n - 1$. It follows immediately that there exists $V^\Delta \geq 0$ to satisfy (24). In what follows, fixing V_L, let us denote the largest V^Δ that solves (24) by $V^\Delta(V_L)$.

Step IV We show that there exists $V_L \in [0,1]$ to satisfy (19), i.e.,

$$\sum_{i\in\mathcal{N}_1} \frac{\delta_i}{1-\delta_i}\left[p_i V_L - c_i\left(\tilde{f}_i^{-1}(Yp_i)\right)\right] + \left(k - |\mathcal{N}_1|\right)V^\Delta + V_L = 1, \tag{25}$$

where $V^\Delta = V^\Delta(V_L)$, $Y = Y(V^\Delta, V_L)$, and $p_i = p_i(Y, V^\Delta, V_L)$ for $i \in \mathcal{N}$, as defined above.

Note that the left-hand side of (25) is always nonnegative; moreover, it is evident that the left-hand side is no less than 1 when $V_L = 1$. To conclude the proof, it suffices to show that $\lim_{V_L \searrow 0} V^\Delta(V_L) = 0$, from which we can conclude that the left-hand side of (25) approaches 0 as $V_L \searrow 0$.

Suppose, to the contrary, that $\limsup_{V_L\searrow 0} V^\Delta(V_L) > 0$. It can then be verified that the following strict inequality holds as $V_L \searrow 0$ and $V^\Delta \to \limsup_{V_L\searrow 0} V^\Delta(V_L)$:

$$V^\Delta(1-p_i) < \frac{V^\Delta}{\delta_i} - p_i(V_L + V^\Delta) + c_i\left(\tilde{f}_i^{-1}(Yp_i)\right),\ \forall\, i \in \mathcal{N}.$$

Recall that $\mathcal{N}_2$ is nonempty by definition. That is, there exists some agent $j \in \mathcal{N}_2$. By (17), we have that

$$V^\Delta(1-p_j) \geq \frac{V^\Delta}{\delta_j} - p_j(V_L + V^\Delta) + c_j\left(\tilde{f}_j^{-1}(Yp_j)\right).$$

A contradiction. ■

Proof of Theorem 2

Proof. We first show that the optimum can be achieved by setting $k = 1$. It suffices to show that for each $(\boldsymbol{\alpha}, \boldsymbol{\beta}, k)$ and a resulting equilibrium, there exists $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}})$ such that $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, 1)$ induces the same equilibrium effort profile $\boldsymbol{x}$ and recognition probabilities $\boldsymbol{p}$.

By (1), (2), and (9), we have

$$p_i = \frac{\alpha_i f_i(x_i) + \beta_i}{\sum_{j\in\mathcal{N}}\left[\alpha_j f_j(x_j) + \beta_j\right]},$$

and

$$c_i'(x_i)\frac{\alpha_i f_i(x_i) + \beta_i}{\alpha_i f_i'(x_i)} \geq p_i(1-p_i)\left(V_L + \frac{(1-p_i-\mu_i)V^\Delta}{1-p_i}\right), \tag{26}$$

with equality holding if $x_i > 0$.

We construct $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}})$ as follows. For $x_i = 0$, we set $(\hat{\alpha}_i, \hat{\beta}_i) = (0, p_i)$. For $x_i > 0$, note by (13) that we have that

$$1 = \sum_{i\in\mathcal{N}_1}(\delta_i v_i) + (k - |\mathcal{N}_1|)V^\Delta + V_L \geq V^\Delta + V_L, \tag{27}$$

where the inequality follows from $v_i \geq 0$ and $|\mathcal{N}_1| \leq k-1$. Combining (26) and (27) yields

$$\frac{c_i'(x_i)f_i(x_i)}{f_i'(x_i)} \leq c_i'(x_i)\frac{\alpha_i f_i(x_i)+\beta_i}{\alpha_i f_i'(x_i)} = p_i(1-p_i)\left(V_L + \frac{(1-p_i-\mu_i)V^{\Delta}}{1-p_i}\right) \leq p_i(1-p_i).$$

Define $\hat{\theta}_i := p_i(1-p_i)f_i'(x_i)/c_i'(x_i) - f_i(x_i)$. The above inequality indicates $\hat{\theta}_i \geq 0$. Set

$$\left(\hat{\alpha}_i, \hat{\beta}_i\right) := \left(\frac{p_i}{f_i(x_i)+\hat{\theta}_i}, \hat{\alpha}_i\hat{\theta}_i\right). \tag{28}$$

It remains to verify that $(\boldsymbol{x}, \boldsymbol{p})$ constitutes the unique equilibrium effort profile and recognition probabilities under $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, 1)$. When $k=1$, the game degenerates to a standard static contest with prize value of 1. It suffices to show that the equilibrium recognition probability p_i satisfies

$$p_i = \frac{\hat{\alpha}_i f_i(x_i) + \hat{\beta}_i}{\sum_{j\in\mathcal{N}}\left[\hat{\alpha}_j f_j(x_j) + \hat{\beta}_j\right]}, \tag{29}$$

and x_i solves

$$\max_{x_i\geq 0} \frac{\hat{\alpha}_i f_i(x_i) + \hat{\beta}_i}{\sum_{j\in\mathcal{N}}\left[\hat{\alpha}_j f_j(x_j) + \hat{\beta}_j\right]} - c_i(x_i). \tag{30}$$

Note that $p_i = \hat{\alpha}_i f_i(x_i) + \hat{\beta}_i$ for all $i \in \mathcal{N}$ by construction (see, e.g., (28)). Therefore, $\sum_{j\in\mathcal{N}}(\hat{\alpha}_j f_j(x_j) + \hat{\beta}_j) = \sum_{j\in\mathcal{N}} p_j = 1$, which implies (29).

Next, we verify that x_i solves the maximization problem (30). For agent $i \in \mathcal{N}$ with $x_i = 0$, it is evident that choosing $x_i = 0$ dominates $x_i > 0$ under $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, 1)$ because $\hat{\alpha}_i = 0$. For agent $i \in \mathcal{N}$ with $x_i > 0$, by (28), we have that

$$c_i'(x_i)\frac{\hat{\alpha}_i f_i(x_i) + \hat{\beta}_i}{\hat{\alpha}_i f_i'(x_i)} = c_i'(x_i)\frac{f_i(x_i)+\hat{\theta}_i}{f_i'(x_i)} = p_i(1-p_i),$$

which is exactly the first-order condition for the maximization problem (30).

The above analysis shows that the optimum can be achieved by $k=1$, in which case the game reduces to a standard static contest. By Theorem 2 in Fu and Wu (2020), the optimum can be achieved by choosing multiplicative biases $\boldsymbol{\alpha}$ only and setting headstart $\boldsymbol{\beta}$ to zero under Assumption 2. ■

Proof of Theorem 3

Proof. It suffices to show that fixing an arbitrary equilibrium effort profile $\boldsymbol{x}^*$ and the resulting recognition probability profile $\boldsymbol{p}^*$ under some voting rule k and contest rule $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, the designer can modify the contest rule under the less inclusive voting rule $k-1$ to induce the same equilibrium effort profile $\boldsymbol{x}^*$ and recognition probability profile $\boldsymbol{p}^*$.

Plugging (11) into (12), we have that

$$\sum_{i\in\mathcal{N}_2}\left[\left(\frac{1}{\delta_i}-p_i^*\right)V^\Delta - p_i^*V_L + c_i(x_i^*)\right] + \sum_{i\in\mathcal{N}_1}(1-p_i^*) = (k-1)V^\Delta. \tag{31}$$

Recall by (19), we have that

$$\sum_{i\in\mathcal{N}_1}\frac{\delta_i}{1-\delta_i}\left[p_i^*V_L - c_i(x_i^*)\right] + \big(k-|\mathcal{N}_1|\big)V^\Delta + V_L = 1. \tag{32}$$

Note that holding fixed $(\boldsymbol{x}^*, \boldsymbol{p}^*)$, we can adjust the contest rule to satisfy the above two equilibrium conditions as the voting rule k varies, which gives a new pair (V^Δ, V_L). To prove the theorem, it remains to verify the following first-order condition under the less inclusive voting rule $k-1$ and the new pair (V^Δ, V_L):

$$\frac{c_i'(x_i^*)f_i(x_i^*)}{f_i'(x_i^*)} \le p_i^*(1-p_i^*)\left[V_L + V^\Delta - \frac{\mu_i}{1-p_i^*}V^\Delta\right], \forall\, i \in \mathcal{N}.$$

Evidently, it suffices to show that the effective prize spread, $V_L + V^\Delta - \frac{\mu_i}{1-p_i^*}V^\Delta$, is nonincreasing in k.

We treat k as a continuous variable. Clearly, μ_i, V^Δ, and V_L are all continuous in k. Moreover, for all but finitely many values of k, the sets $\mathcal{N}_1$, $\mathcal{N}_2$, and $\mathcal{N}_3$ remain unchanged in a neighborhood of k, which indicates that μ_i, V^Δ, and V_L are differentiable with respect to k. Therefore, it suffices to show that the derivative of the effective prize spread, $V_L + V^\Delta - \frac{\mu_i}{1-p_i^*}V^\Delta$, with respect to k is nonpositive whenever it is differentiable. Taking the derivatives of (31) and (32) with respect to k yields that

$$\frac{dV_L}{dk} = -\frac{(\mathcal{B}+\mathcal{D})V^\Delta}{\mathcal{A}\mathcal{D}+\mathcal{B}\mathcal{C}} \text{ and } \frac{dV^\Delta}{dk} = -\frac{(\mathcal{C}-\mathcal{A})V^\Delta}{\mathcal{A}\mathcal{D}+\mathcal{B}\mathcal{C}},$$

where

$$\begin{aligned}
\mathcal{A} &:= 1 + \sum_{i\in\mathcal{N}_1}\frac{p_i^*\delta_i}{1-\delta_i} > 0,\\
\mathcal{B} &:= k - |\mathcal{N}_1| > 0,\\
\mathcal{C} &:= \sum_{i\in\mathcal{N}_2} p_i^* > 0,
\end{aligned}$$

$$\mathcal{D} := \sum_{i\in\mathcal{N}_1}(1-p_i^*) + \sum_{i\in\mathcal{N}_2}\left(\frac{1}{\delta_i} - p_i^*\right) - (k-1) > 0.$$

Evidently, $\mathcal{AD}+\mathcal{BC}>0$ and $\mathcal{B}+\mathcal{D}>0$. Moreover, we have that

$$\mathcal{C}-\mathcal{A} = \sum_{i\in\mathcal{N}_2} p_i^* - 1 - \sum_{i\in\mathcal{N}_1}\frac{p_i^*\delta_i}{1-\delta_i} \le -\sum_{i\in\mathcal{N}_1}\frac{p_i^*\delta_i}{1-\delta_i} \le 0.$$

Therefore, we have that

$$\frac{dV_L}{dk} = -\frac{(\mathcal{B}+\mathcal{D})V^\Delta}{\mathcal{AD}+\mathcal{BC}} \le 0 \text{ and } \frac{dV^\Delta}{dk} = -\frac{(\mathcal{C}-\mathcal{A})V^\Delta}{\mathcal{AD}+\mathcal{BC}} \ge 0. \tag{33}$$

We consider the following three cases.

Case I: $i\in\mathcal{N}_1$. By (11), $\mu_i = 1-p_i^*$ and the effective prize spread is V_L. We can then conclude from (33) that V_L is non-increasing in k.

Case II: $i\in\mathcal{N}_2$. By (11), the effective prize spread is

$$\begin{aligned} V_L + V^\Delta - \frac{\mu_i}{1-p_i^*}V^\Delta =& V_L + V^\Delta - \frac{V^\Delta}{\delta_i(1-p_i^*)} + \frac{p_i^*}{1-p_i^*}(V_L+V^\Delta) - c_i(x_i^*) \\ =& \frac{V_L}{1-p_i^*} - \frac{(1-\delta_i)V^\Delta}{\delta_i(1-p_i^*)} - c_i(x_i^*). \end{aligned}$$

Carrying out the algebra, we can obtain that

$$\frac{d}{dk}\left(V_L + V^\Delta - \frac{\mu_i}{1-p_i^*}V^\Delta\right) = \frac{1}{1-p_i^*}\times\frac{dV_L}{dk} - \frac{1-\delta_i}{\delta_i(1-p_i^*)}\times\frac{dV^\Delta}{dk} \le 0.$$

Case III: $i\in\mathcal{N}_3$. By (11), $\mu_i = 0$ and the effective prize spread is V_L+V^Δ. By (33), we have that

$$\frac{d(V_L+V^\Delta)}{dk} = \frac{(\mathcal{B}+\mathcal{C}+\mathcal{D}-\mathcal{A})V^\Delta}{\mathcal{AD}+\mathcal{BC}}.$$

It remains to prove

$$\mathcal{B}+\mathcal{C}+\mathcal{D}-\mathcal{A} \ge 0.$$

Carrying out the algebra, we have that

$$\mathcal{B}+\mathcal{C}+\mathcal{D}-\mathcal{A} = k - |\mathcal{N}_1| + \sum_{i\in\mathcal{N}_2}p_i^* + \sum_{i\in\mathcal{N}_1}(1-p_i^*) + \sum_{i\in\mathcal{N}_2}\left(\frac{1}{\delta_i}-p_i^*\right) - (k-1) - 1 - \sum_{i\in\mathcal{N}_1}\frac{p_i^*\delta_i}{1-\delta_i}$$

$$
\begin{aligned}
&= \sum_{i \in \mathcal{N}_2} \frac{1}{\delta_i} - \sum_{i \in \mathcal{N}_1} \frac{p_i^*}{1 - \delta_i} \\
&\geq 2|\mathcal{N}_2| - 2 \sum_{i \in \mathcal{N}_1} p_i^* \geq 0,
\end{aligned}
$$

where the first inequality follows from $\delta_i \leq \frac{1}{2}$. This concludes the proof. ■

Appendix B: Derivation for Examples

Derivation for Equilibria in Example 1

First, consider the case of $k = 1$. The game reduces to a static Tullock contest. Let $Y := \sum_{i\in\mathcal{N}} \eta_i x_i$. The equilibrium conditions can be derived as

$$Y = 1 - p_i,$$

from which we can solve for the equilibrium aggregate effort Y, the equilibrium recognition probabilities $\boldsymbol{p} = (p_1, p_2, p_3, p_4)$, and the equilibrium efforts $\boldsymbol{x} = (x_1, x_2, x_3, x_4)$ as follows:

$$Y = \frac{3}{4},$$

$$p_i = 1 - Y = \frac{1}{4},\ \forall i \in \mathcal{N},$$

and

$$\boldsymbol{x} = Y\boldsymbol{p} \oslash \boldsymbol{\eta} = \left(\frac{3}{16}, \frac{15}{16}, \frac{15}{16}, \frac{15}{16}\right).$$

Next, consider the case of $k = 2$. The equilibrium conditions in the proof of Theorem 1—i.e., conditions (14), (15), (16), (17), (18), and (19)—for this example can be expressed as follows:

$$Yp_i = \eta_i x_i,$$

$$\sum_{i\in\mathcal{N}} p_i = 1,$$

$$Y = (1 - p_i)(V_L + V^\Delta) - \mu_i V^\Delta,$$

$$\mu_i = \frac{1}{\delta_i} - p_i - \frac{p_i V_L - Y p_i}{V^\Delta},$$

$$\sum_{i\in\mathcal{N}} \mu_i = 1,$$

$$\frac{1}{9}\left[p_1 V_L - Y p_1\right] + V_L + V^\Delta = 1.$$

It can be verified that $\boldsymbol{p} = (0.2322, 0.2559, 0.2559, 0.2559)$, $\boldsymbol{x} = (0.1711, 0.9433, 0.9433, 0.9433)$, $\boldsymbol{\mu} = (0.7678, 0.0774, 0.0774, 0.0774)$, $V_L = 0.9600$, and $V^\Delta = 0.0342$ constitute an SSPE of the game. The equilibria for the cases of $k = 3$ and $k = 4$ can be similarly verified.

Derivation for the Optimal Recognition Mechanism in Example 2

We demonstrate the optimality of $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$ in Example 2. When the designer sufficiently cares about the profile of agents' recognition probabilities—i.e., when $\lambda \gg 1/c$—the optimal equilibrium winning probability profile must be $\boldsymbol{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and the designer's payoff at $\boldsymbol{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ reduces to $\Lambda = x_1 + x_2 + x_3$. When c is sufficiently small, agent 3 is excessively strong and the designer's payoff is mainly determined by x_3. Therefore, it suffices to show that $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$ maximizes x_3 among all rules $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ that induce $\boldsymbol{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

Fix $\boldsymbol{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. We first rewrite the equilibrium conditions in the proof of Theorem 1—i.e., conditions (14)-(19). Evidently, condition (15) is satisfied and condition (14) becomes

$$\alpha_i^* x_i + \beta_i^* = \frac{Y}{3}, \ \forall\, i \in \{1, 2, 3\}. \tag{34}$$

Next, consider condition (16). The condition holds with equality for $x_i > 0$. Further, if $x_i = 0$ for some $i \in \mathcal{N}$ and the strict inequality holds, we can increase α_i until the equality holds and at the same time keep unchanged the equilibrium effort profile $\boldsymbol{x}$ and recognition probabilities $\boldsymbol{p}$. Therefore, we can assume that equality holds for all agents and the condition becomes

$$\frac{Y c_i}{\alpha_i^*} = \frac{2(V_L + V^\Delta)}{3} - \mu_i V^\Delta, \ \forall\, i \in \{1, 2, 3\}. \tag{35}$$

Substituting (35) into (34) yields

$$3 c_i x_i \leq \frac{2(V_L + V^\Delta)}{3} - \mu_i V^\Delta, \ \forall\, i \in \{1, 2, 3\}, \tag{36}$$

with equality holding if $\beta_i^* = 0$. To establish the optimality of headstarts, it suffices to show that the inequality is strict for at least one agent.

Conditions (17), (18), and (19) are

$$\mu_i = \begin{cases} \frac{2}{3} \leq \frac{1}{\delta_i} - \frac{1}{3} - \frac{V_L}{3V^\Delta} + \frac{c_i x_i}{V^\Delta}, & i \in \mathcal{N}_1, \\ \frac{1}{\delta_i} - \frac{1}{3} - \frac{V_L}{3V^\Delta} + \frac{c_i x_i}{V^\Delta} \in \left[0, \frac{2}{3}\right], & i \in \mathcal{N}_2, \\ 0 \geq \frac{1}{\delta_i} - \frac{1}{3} - \frac{V_L}{3V^\Delta} + \frac{c_i x_i}{V^\Delta}, & i \in \mathcal{N}_3, \end{cases} \tag{37}$$

$$\mu_1 + \mu_2 + \mu_3 = 1, \tag{38}$$

and

$$\sum_{i \in \mathcal{N}_1} \frac{\delta_i}{1 - \delta_i} \left(\frac{V_L}{3} - c_i x_i \right) + \left(2 - |\mathcal{N}_1|\right) V^\Delta + V_L = 1. \tag{39}$$

Substituting (37) in (36) yields that

$$c_i x_i \leq \begin{cases} \frac{2}{9} V_L, & i \in \mathcal{N}_1, \\ \frac{1}{4}\left[V_L - (\frac{1}{\delta_i} - 1)V^{\Delta}\right], & i \in \mathcal{N}_2, \\ \frac{2}{9}(V_L + V^{\Delta}), & i \in \mathcal{N}_3, \end{cases} \tag{40}$$

from which we can conclude $c_i x_i \leq \frac{2V_L}{9}$ for $i \in \mathcal{N}_1$; together with (39), we can obtain that

$$\sum_{i \in \mathcal{N}_1} \frac{\delta_i}{1-\delta_i} \times \frac{V_L}{9} + \big(2 - |\mathcal{N}_1|\big) V^{\Delta} + V_L \leq 1. \tag{41}$$

In what follows, we will show that $c_3 x_3 \leq \frac{30}{144-\delta_3}$, and the equality holds if and only if $\boldsymbol{\alpha}^* = (\frac{62Y}{35}, \frac{62Y}{37}, \frac{62Yc}{39})$ and $\boldsymbol{\beta}^* = (0, \frac{17Y}{222}, 0)$. Consider the following three cases.

Case I: $\mathbf{3} \in \boldsymbol{\mathcal{N}_1}$. Note that $|\mathcal{N}_1| \leq k - 1 = 1$, we have that $\mathcal{N}_1 = \{3\}$. By (41), we can obtain that

$$\left[1 + \frac{\delta_3}{9(1-\delta_3)}\right] V_L + V^{\Delta} \leq 1;$$

together with (39), we can obtain that

$$c_3 x_3 \leq \frac{2V_L}{9} \leq \frac{2(1-\delta_3)}{9 - 8\delta_3} < \frac{30}{144 - \delta_3}.$$

Case II: $\mathbf{3} \in \boldsymbol{\mathcal{N}_2}$. By (37) and (40), we have that

$$0 \leq \frac{1}{\delta_3} - \frac{1}{3} - \frac{V_L}{3V^{\Delta}} + \frac{c_3 x_3}{V^{\Delta}} \leq \frac{1}{\delta_3} - \frac{1}{3} - \frac{V_L}{3V^{\Delta}} + \frac{V_L - (\frac{1}{\delta_3} - 1)V^{\Delta}}{4V^{\Delta}}.$$

Carrying out the algebra, we can obtain that

$$V_L \leq \left(\frac{9}{\delta_3} - 1\right) V^{\Delta} = \frac{35}{4} V^{\Delta}. \tag{42}$$

Further, $3 \notin \mathcal{N}_1$ implies that $\mathcal{N}_1 \in \big\{\{1\}, \{2\}, \emptyset\big\}$, and thus (41) becomes

$$1 \geq \left\{ \begin{array}{ll} \frac{16}{15} V_L + V^{\Delta}, & \text{if } \mathcal{N}_1 = \{1\} \\ \frac{10}{9} V_L + V^{\Delta}, & \text{if } \mathcal{N}_1 = \{2\} \\ V_L + 2V^{\Delta}, & \text{if } \mathcal{N}_1 = \emptyset \end{array} \right\} \geq \frac{16}{15} V_L + V^{\Delta}, \tag{43}$$

where the last inequality follows from (42).

Combining (40), (42), and (43), we have that

$$c_3x_3 \leq \frac{1}{4}\left[V_L - (\frac{1}{\delta_3} - 1)V^\Delta\right] \leq \frac{30}{144-\delta_3} = \frac{13}{62}.$$

Note that equality holds in condition (40) if and only if $\beta_3^* = 0$. Further, equality holds in condition (42) only if $\mu_3 = 0$. Last, equality holds in condition (43) if and only if $\mathcal{N}_1 = \{1\}$ and $\beta_1^* = 0$.

Because $\mathcal{N}_1 = \{1\}$ and $\mu_3 = 0$, we have that $\mu_1 = \frac{2}{3}$ from (37); together with (38), we have $\mu_2 = \frac{1}{3}$. Moreover, by (37), we can conclude $2 \in \mathcal{N}_2$, which implies that $\mathcal{N}_2 = \{2, 3\}$ and $\mathcal{N}_3 = \emptyset$.

Combining (42) and (43) (recall that equality holds in these two conditions), we can obtain $V_L = \frac{105}{124}$ and $V^\Delta = \frac{3}{31}$; together with (40), we have $x_1 = \frac{2V_L}{9} = \frac{35}{186}$. Substituting $\mu_2 = \frac{1}{3}$, $V_L = \frac{105}{124}$ and $V^\Delta = \frac{3}{31}$ in (37), we can obtain that $x_2 = \frac{V_L - 4V^\Delta}{3} = \frac{19}{124}$.

Last, we solve for $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*)$. Recall that $\beta_i^* = 0$ for $i \in \{1, 3\}$. Therefore, $\alpha_i^* = \frac{Y}{3x_i}$ from (34). For $i = 2$, we have $x_2 = \frac{19}{124}$. Further, by (35), we have $\frac{Y}{\alpha_2^*} = \frac{2V_L + V^\Delta}{3} = \frac{37}{62}$, which implies that $\alpha_2^* = \frac{62Y}{37}$; together with (34), we can conclude $\beta_2^* = \frac{Y}{3} - \alpha_2^* x_2 = \frac{17Y}{222}$.

In summary, the equality holds in $c_3x_3 \leq \frac{30}{144-\delta_3}$ if and only if $\boldsymbol{\alpha}^* = (\frac{62Y}{35}, \frac{62Y}{37}, \frac{62Yc}{39})$ and $\boldsymbol{\beta}^* = (0, \frac{17Y}{222}, 0)$, under which the equilibrium is $\boldsymbol{x} = (\frac{35}{186}, \frac{19}{124}, \frac{39}{186c})$, $\boldsymbol{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, $\boldsymbol{\mu} = (\frac{2}{3}, \frac{1}{3}, 0)$, $V_L = \frac{105}{124}$, and $V^\Delta = \frac{3}{31}$.

Case III: 3 ∈ $\boldsymbol{\mathcal{N}_3}$. Condition (37), together with the postulated $3 \in \mathcal{N}_3$, implies that $\mu_3 = 0$. Analogous to derivation of (42), we can obtain that

$$V_L > \left(\frac{9}{\delta_3} - 1\right)V^\Delta = \frac{35}{4}V^\Delta. \tag{44}$$

Suppose $\mathcal{N}_1 \neq \emptyset$. By (43), we have that

$$1 \geq \frac{16V_L}{15} + V^\Delta. \tag{45}$$

Combining (36), (44), and (45) yields that

$$c_3x_3 \leq \frac{2(V_L + V^\Delta)}{9} < \frac{30}{144-\delta_3}.$$

Next, suppose $\mathcal{N}_1 = \emptyset$; together with $3 \in \mathcal{N}_3$ and $k = 2$, we can conclude $\mathcal{N}_2 = \{1, 2\}$. It follows from (39) that

$$V_L + 2V^\Delta = 1. \tag{46}$$

Recall $\mu_3 = 0$. Combining (37), (38), and (40), we can obtain that

$$1 = \mu_1 + \mu_2 = \frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{2}{3} - \frac{2V_L}{3V^\Delta} + \frac{x_1 + x_2}{V^\Delta} \leq 4 - \frac{2V_L}{3V^\Delta} + \frac{V_L}{2V^\Delta} - \frac{2}{3},$$

which in turn implies that

$$V_L \leq 14V^\Delta. \tag{47}$$

Therefore,

$$c_3 x_3 \leq \frac{2(V_L + V^\Delta)}{9} \leq \frac{5}{24} < \frac{30}{144 - \delta_3},$$

where the first inequality follows from (36) and the second inequality from (46) and (47).

Derivation for the Optimal Voting Rule in Example 3

By Theorem 2, the optimum can be achieved by setting $k = 1$. It remains to show that the optimum can be achieved by setting $k = 5$, but not $k = 4$. Evidently, when λ is sufficiently large, the optimum requires that $\boldsymbol{p} = \tilde{\boldsymbol{p}}$. Moreover, when γ is sufficiently large, each agent i's equilibrium effort x_i cannot exceed $\tilde{x}_i$. Therefore, it suffices to show that fixing $\boldsymbol{p} = \tilde{\boldsymbol{p}}$, the equilibrium effort is $\boldsymbol{x} = \tilde{\boldsymbol{x}}$ at $k = 5$, and the designer cannot induce $\boldsymbol{x} = \tilde{\boldsymbol{x}}$ at $k = 4$.

When $k = 5$, it can be verified that $\mathcal{N}_1 = \{1, 2, 3\}$, $\mathcal{N}_2 = \{4, 5, 6\}$, and $\mathcal{N}_3 = \{7\}$. Moreover, the equilibrium effort is $\tilde{\boldsymbol{x}}$, and equilibrium winning probability is $\tilde{\boldsymbol{p}}$. In this case, agent 7's effective prize spread is $V_L + V^\Delta = 0.8399$, and his first-order condition holds with equality:

$$r\tilde{x}_7 c_7'(\tilde{x}_7) = (V_L + V^\Delta)\tilde{p}_7(1 - \tilde{p}_7).$$

Next, we show that when $k = 4$, the designer cannot induce $\boldsymbol{p} = \tilde{\boldsymbol{p}}$ and $\boldsymbol{x} = \tilde{\boldsymbol{x}}$ simultaneously. In fact, fixing $\boldsymbol{p} = \tilde{\boldsymbol{p}}$ and $\boldsymbol{x} = \tilde{\boldsymbol{x}}$, by (17)-(19), we have that $V_L = 0.7439$ and $V^\Delta = 0.0669$, with $V_L + V^\Delta < 0.8399$. However, agent 7's first-order condition requires that

$$r\tilde{x}_7^r = \tilde{x}_7 c_7'(\tilde{x}_7) = 0.8399 \times \tilde{p}_7(1 - \tilde{p}_7) \leq (V_L + V^\Delta)\tilde{p}_7(1 - \tilde{p}_7).$$

A contradiction.